



## PRACTICAL APPLICATION OF GEOMETRIC CONSTRUCTIONS IN SOLVING POSITIONAL TASKS IN THE PERSPECTIVE

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### ABSTRACT

The perspective image from the execution is used in practice in architecture, fine art, in design and reconstruction works. Checking the positional-metric relations between geometric figures in perspective requires a person to have a strong spatial imagination and thinking, logical thinking. There are certain difficulties in teaching such questions and understanding them by students. This article describes the content of positional questions on which perspective is taught, their types and practical significance, the place that perspective occupies in the execution of the image. The article mainly discusses methods for solving positional problems using geometric constructions.

### KEYWORDS

Central projection, perspective, perspective image, positional problem, metric problem, point of incidence of a straight line, fine art, painting, auxiliary triangles, adjacent rectangles, horizontal, parallel lines, geometric construction, spatial imagination, thinking, logical thinking.

### INTRODUCTION

This article is written in order to introduce effective methods of preparation and on the condition of taking into account the changing needs of our country's labor market, individual, state and society, based on the needs of employees of the system of higher education and innovation, pre-school and school education

ministries, and prepares future teachers of fine arts and drawing, moreover, in order to ensure its implementation, based on the content of the defined priority tasks of the "On Education" dated September 23, 2020 URL-637, President of the Republic of Uzbekistan "On the Development Strategy of the New Uzbekistan



for 2022-2026" dated January 28, 2022 No. PD-60, "On approval of the concept of development of science until 2030" No. 6097 of October 29, 2020, "Development of the fields of education and science in the new development period of Uzbekistan on measures" No. PF-6108 of November 6, 2020, "National program for the development of public education in 2022-2026" No. PF-134 of May 11, 2022 Decrees and PD-4884 of November 6, 2020, "On additional measures to further improve the education system" dated November 6, 2020, "Research in the field of public education on measures to support the activity and introduce a continuous professional development system" Resolutions PD-4963 of January 25, 2021, and the Cabinet of Ministers of January 17, 2022 No. 25 "On approval of the regulation on the procedure for the organization of the system of continuous professional development of public education employees" and the Minister of Public Education of April 20, 2022 "State Education Order No. 121 on approval of requirements".

Every parent and country are dying for the future of their children. He tries to create sufficient conditions for them. In our country, like many developed countries, conditions for the education of young people have been created and are being improved. As the President of the Republic of Uzbekistan Sh.M. Mirziyoyev said, "Of course, only you, our dear young people, who have mastered modern knowledge and skills, think independently, and always live with a sense of belonging to the fate of the country, will boldly go out into the field and become the leaders of life today. You are capable of solving the tasks that he sets before us. <sup>1</sup>"

In one of his speeches, President Sh.M. Mirziyoyev emphasized that "... further development of not only academic science, but also science in higher educational institutions is an important task. <sup>2</sup>" The organization of the educational process and the training of intellectually capable personnel, combining the materials of historical sources and new modern technical development innovations, remain a requirement of the present day. To meet these demands, great positive changes are taking place in education.

### LITERATURE REVIEW

The introduction of advanced pedagogical technologies into education and its effective use will be a worthy response to the above requirements. We will achieve this through innovations in education. Interactive strategies of education are among the innovations in education. According to K. Angelevsky, "... all countries are trying to introduce as many innovations as possible to education. Today's news requires an organized, planned, public approach to them. News is a long-term investment for the future.

In order to arouse interest in innovation, to educate a person who strives to create new things, education itself should be rich in innovations, and the spirit and atmosphere of creativity should prevail in it. <sup>3</sup>"

Along with many other sciences, the science of drawing geometry is one of the necessary sciences that serve to meet the needs of society, mathematics, construction, engineering, visual arts and other fields. If mathematics is mental gymnastics, drawing

on the results of 2016 and prospects for 2017. - T.: "Uzbekistan", 2017. Page 46.

<sup>3</sup> Махкамова С. Педагогические технологии: понятия, принципы и методы внедрения: Сб. материалов из Интернета./ Сост. С.Махкамова. – Ташкент, 2003. – С.26.

<sup>1</sup> The speech of the President of the Republic of Uzbekistan Sh. Mirziyoyev at the 4th congress of the "Kamolot" youth social movement.

<sup>2</sup> Mirziyoyev Sh.M. Critical analysis, strict discipline and personal responsibility should be the daily rules of every leader's activity. Speech of the Cabinet of Ministers in 2016



geometry is a science that develops human spatial imagination.

Let's talk about the research work carried out by scholars and specialist scientists from drawing geometry, engineering graphics, perspective, drawing.

Yu. Kyrgyzboev, R. Khorunov, I. Rakhmanov, Sh. Murodov, P. Adilov, D. Kochkarova, Shomurodov, B. Khaitov, M. N. Makarova, S. A. Soloviev, N. V. Kaygorodsev, P.A. Florensky, Khorla Teodoriu, Marcos Mateu-Mestrelar, P. Adilov, Sh. Abdurakhmanov, A. Umronkhojaev, E. Roziyev, K. Zoirov, A. Khamrakulov, S. Saidaliyev on the methodology of teaching it, J. Yodgorovlar, A. Khamrakulov, Sh. Dilshodbekov, Sh. Dilshodbekov, Sh. Muslimov and others have carried out scientific research on the introduction of innovative educational technologies, but it is precisely in these works that we focus on teaching perspective science the content of the raised problem has not been sufficiently researched.

One of the major branches of drawing geometry is perspective, which serves the development of architecture and visual arts (perspective is taught as a separate subject in some educational institutions). The perspective image is made on the basis of the central projection method and it is monoprojection. Because of this, students have some difficulty in moving from space to plane drawing or, conversely, from plane drawing to space.

Among the works of many famous oriental scholars in the middle of the 12th century, Gerardo Ali Hasan ibn al-Haysam al-Basri of Cremona wrote several books, among them his "Kitab al-Manoziri" under the titles "De Aspectibus" and "Perspectiva". translated into Latin under. <sup>4</sup>

These books were of particular importance in the development of scientists such as Bacon and Peckham, who were seriously engaged in the science of optics in Europe in the 13th century. The famous naturalist Roger Bacon (1214-1292), who grew up under the influence of Al-Haytham's books, conducted many experiments on light rays with the help of his camera obscura and the use of lenses of various shapes. He developed the theory of glasses. By the end of the 13th century, production of convex and then concave glasses was started in Italy. <sup>5</sup>

The Polish architect Vitello (1225 - 1280) named his ten books on optics "Perspectiva" (Latin: "seeing inside"). In writing this book, he made good use of al-Haytham's collection of works on optics ("Opticae thesaurus")<sup>6</sup>. For example, al-Haysam corrected the Pythagorean misconception about vision held by many opticians before him: "Optical image is formed by light rays radiating from visible objects. it will be," he said<sup>7</sup>. It can be seen that many encyclopedic scholars served in the formation and development of the science of optics and perspective.

<sup>4</sup> Абдурахмонов Ш. "Перспектива" или атамасининг расмий фанга кириб, унда қарор топиши хусусида. Узлуксиз таълим тизимининг чизма геометрия ва муҳандислик графикаси йўналишида педагог кадрлар тайёрлашнинг илмий-назарий асослари Республика илмий-амалий анжумани материаллари. ТДПУ, 2015. 84-85 бетлар.

<sup>5</sup> Тарасов Л.В., Тарасова А.Н. Беседы о преломлении света /Под ред. В.А. Фабриканта. – М.: Наука, 1982.

<sup>6</sup> Абдурахмонов Ш. Чизмалар яратишда қўлланилган ҳандасаилми. – Т.: "Fan va texnologiya". 2017-48-bet.

<sup>7</sup> Гамалина Р.Б., Соколова Е.И. К истории геометрической оптики на Ближнем и Среднем Востоке и в Европе в средние века //Сб. Из истории точных наук на средневековом Ближнем и Среднем Востоке. – Т.: 1972. – С.: 72 – 76.



## Research Methodology.

Since the perspective image is considered a monoproduction, the central projection (perspective) of geometric shapes has its own uniqueness compared to the image in the orthogonal projection. That is, a straight line is described by a drop point or a plane drop line. As a result, it becomes difficult to harmoniously imagine and understand the solution of positional and metric problems both in space and in the epur (working situation). That is, it is difficult for students to adapt (get used to) central projection. In order to overcome this difficulty, as a result of organizing the lessons using orthogonal and central projection methods comparison (comparative analysis), the achievement of the purpose of the lesson will be accelerated and the quality will increase.

Now let's turn to the content of positional issues and the existing problems in teaching it. One of the goals of the science of drawing geometry is to determine the interrelationships of points, lines, and flat shapes in geometric figures, i.e. positional and metrical relationships between them.

Depending on the arrangement of geometric figures in the drawing, the geometric problems related to them can be divided into three groups.

1. Positional issues.
2. Metric issues.
3. Constructive issues<sup>8</sup>.

Problems aimed at determining the third geometric shape formed by the mutual situation of two geometric shapes in relation to each other are called positional problems. For example, the intersection of a straight line and a plane (if they intersect) creates a

point. Or, a third geometric shape - a line of intersection (a spatial curve, a straight line, or a spatial broken line) is formed from the intersection of two intersecting surfaces.

We can include the following geometrical problems in the composition of positional problems:

- to determine the line intersecting two planes;
- to determine the intersection, point of a straight line with a plane;
- drawing a straight line perpendicular to the plane;
- to divide the cross section of a straight line into a given ratio;
- determining the intersecting line of two surfaces;
- determining the intersection points of a straight line with a surface, etc<sup>9</sup>.

In perspective, it is always necessary to determine the position of this or that object in space or the location of their elements in relation to the object, and then to make its perspective. That is, it is necessary to know how to solve a positional problem. In drawing, designing and construction work, it is necessary to determine or know the shape, structure, size of geometric shapes in details and objects. In such cases, it is necessary to know how to solve positional problems. For example, a number of positional problems are performed when determining the personal and falling shadows of a building or when constructing an object perspective. Therefore, teaching positional issues together with their practical significance will provide a basis for students to acquire knowledge and skills in the subject at a high quality and effective level. Aspects of the topic related to life are

<sup>8</sup> Муродов Ш., Ҳакимов Л., Одилов П., Шомуродов А., Жумаев М. Чизма геометрия курси. –Т.: «Ўқитувчи», 1988, 3- бет.

<sup>9</sup> Valiyev A.N. Perspektiva. -Т.: “Voriz-nashriyot”, 2012, 91-бет.





mastered. It is also effective to solve perspective positional problems based on simple geometric constructions. It is necessary to approach some of the geometric constructions as a method, to reveal its practical importance in solving positional problems, to achieve the possibilities of solving positional problems in convenient ways, and to convey to readers and students their useful applications in drawing. Now we will analyze some situations that reveal the solution of positional issues and their practical importance.

### ANALYSIS AND RESULTS

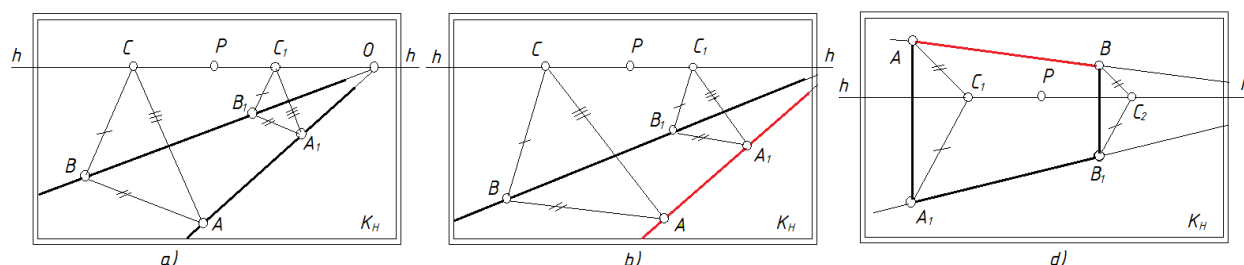
When drawing a perspective image (picture) of various objects, it is sometimes necessary to use conventions and rules. This is certainly related to the solution of positional situations of simple geometric shapes<sup>10</sup>. Therefore, developing examples of solving positional problems using some geometric constructions is an urgent issue today.

Example 1. Method of similar triangles. This method is based on well-known geometrical situations. The center of similarity  $O$  of the ends of two similar triangles lies on one ray (a straight line) and its corresponding sides are parallel to each other (Fig. 1, a). We use this rule to draw parallel straight lines.

Fig. 1, b shows the execution of the edge boundary lines of the road through the field in a mutually parallel

situation. Here is one side of the roadside. It is necessary to pass the second edge of the road parallel to it through point  $A$ . For this, a triangle  $ABC$  is drawn through the given point  $A$ , the second end is the point  $B$  on the given edge of the road, and the third end is the point  $C$  on the horizon line. Then point  $B_1$  is selected at an arbitrary location of the given road edge, and a line parallel to  $BC$  is drawn from it, and point  $C_1$  is determined, intersecting it with the horizon line. Then parallel lines are drawn from point  $B_1$  to  $AB$ , from point  $C_1$  to  $AC$ , and their intersection point  $A_1$  is determined ( $BC \parallel B_1C_1$ ,  $BA \parallel B_1A_1$ ,  $CA \parallel C_1A_1$ ). Point  $A_1$  defined here is the point perspective of the other end of the road. If we connect the points  $A$  and  $A_1$ , we will have the perspective of the other side of the road. As you can see from the drawing, all the straight lines here are horizontal straight lines lying in the object plane  $H$ .

We can apply this method to parallel straight lines lying in the vertical plane (Fig. 1, d). In this drawing, we are asked to construct the perspective of a right-angled rectangle  $A_1ABB_1$  with height  $AA_1$  and length  $A_1B_1$ . To determine the perspective of its side  $AB$ , an arbitrary point  $C_1$  on the horizon is determined and a triangle  $A_1AC_1$  is formed, and a right-angled rectangle perspective  $A_1ABB_1$  is formed using the method of similar triangles described above (Fig. 1, d).



<sup>10</sup> Макарова М.Н. Практическая перспектива. -Москва: "Академический Проект", 2004, 75-стр.



Figure 1

Let's look at another example. Let it be asked to draw a bank boundary through a point A corresponding to its upper bank (embankment boundary) parallel to the straight line given as the water's edge (bank start boundary). The solution to this problem is also done using the method of similar triangles. When we look at the perspective image, the parallel horizontal straight lines determine the inclined (flow) plane, that is, the embankment (coastal plain) is depicted in an oblique position (Fig. 2, a).

Now, let the cross-section AN and its projection AN<sub>1</sub> in the reverse situation be given in the picture (Fig. 2, b). It is necessary to draw a straight line BE parallel to AN by the method of similar triangles through an arbitrary

point B in space. The straight lines connecting the similar triangles in this example are perpendicular to the parallel straight lines drawn through the point of incidence  $a'\infty$ .

First, we draw a triangle through points A, B and  $a'\infty$ . Then we pass the second triangle similar to the first triangle through the point N and determine the point E. Connecting points B and E, we get the perspective (direction) of the line BE parallel to AN. In our example, lines AN and BE are continuations, that is, their point of descent is outside the map  $a\infty$ . The continuation of the vertical straight line drawn from the point  $a'\infty$  also goes to  $a\infty$ .

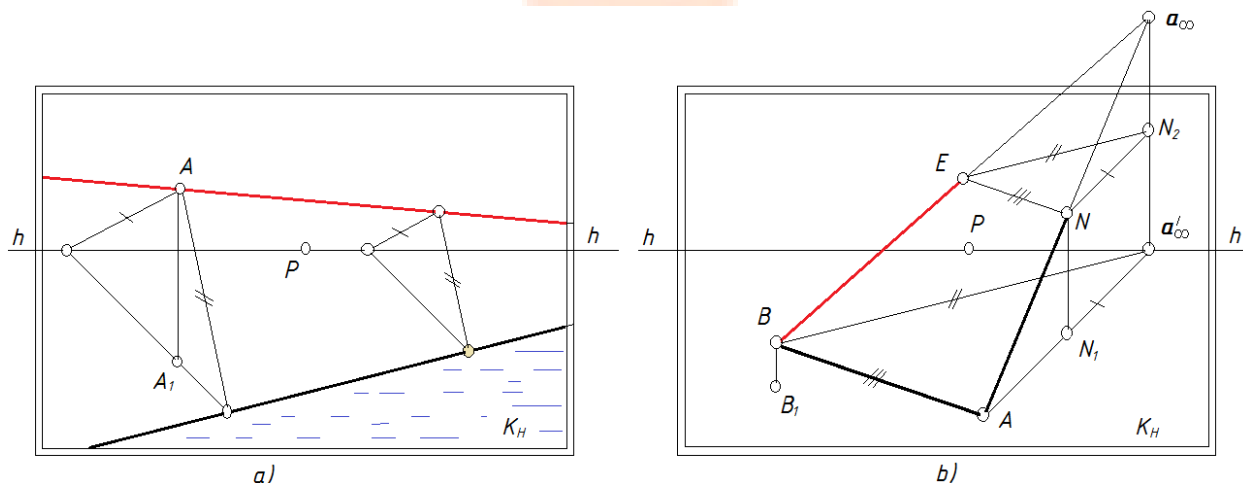


Figure 2

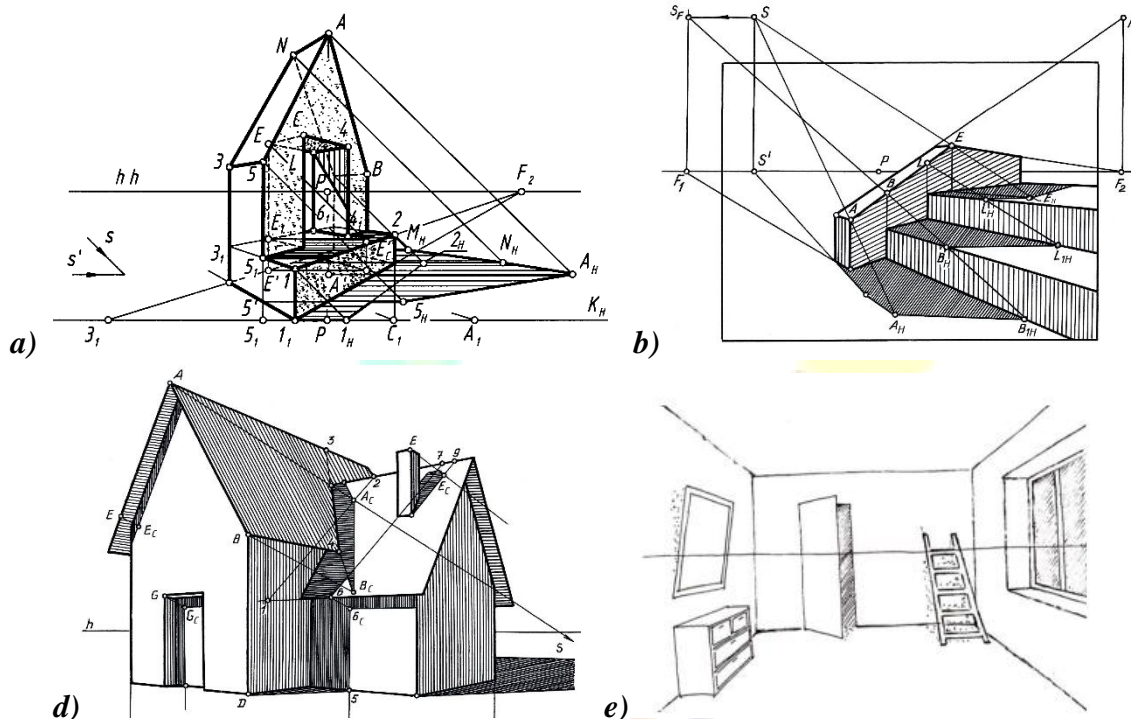
Thus, using the method of similar triangles, we can make a perspective of mutually parallel straight lines that define horizontal, vertical and horizontal planes.

For example, the edges A5 and N3 parallel to each other ( $A5 \parallel N3$ , Fig. 3, a) of the architectural object given in Fig. 3, a fragment of the line parallel to the edge AE of the given stair parapet (Fig. 3, b), The method of similar triangles can be practically used in drawing the



perspective of the line parallel to the edge of the sloping roof of the building (a part of it is visible in the

drawing, Fig. 3, d) or the perspective of the ladder given in Fig. 3, e).



**Figure 3**

Example 2. Adjacent vertical rectangle method. First, let's look at the geometric basis of this construction. If we cross the diagonals of two adjacent rectangles, their intersection points ( $A_2 \cap C_1 \Rightarrow 3$ ,  $B_2 \cap E_1 \Rightarrow 4$ ) lie on one vertical straight line (Fig. 4). Using the above-mentioned geometric basis, the horizontal edge BE of the parallelepiped in Fig. 5 is determined. For this purpose, points 1 and 2 intersecting the horizon line of the vertical lines drawn from points A and C are determined. Diagonals A2 and C1 are drawn and their intersection point 3 is determined. The vertical line drawn from point 3

intersects the diagonal B2 and gives point 4. Then points 1 and 4 are connected, and point E is determined, which intersects it with a vertical line drawn from point C. BE is the horizontal edge of the cross-sectional parallelepiped (Fig. 5, a).<sup>11</sup>

The perspective of the side ABED of the parallelepiped is constructed in the same way as the side ABEC (Fig. 5, b). As a result, the drawing (perspective) of the parallelepiped will be done correctly (Fig. 5, c).

The above geometric constructions (perspective rules) can be done without using drawing tools, and

<sup>11</sup> Valiyev A.N., Rakhmatova I., Abdumutalibova A. Practical Application of Prospects Rules in Drawing (научно-

методическая статья). Eastern European Scientific Journal (ISSN 2199-7977), Germaniya, №5, 2018, 82-86 стр.



the teacher should show the students this in a practical way.

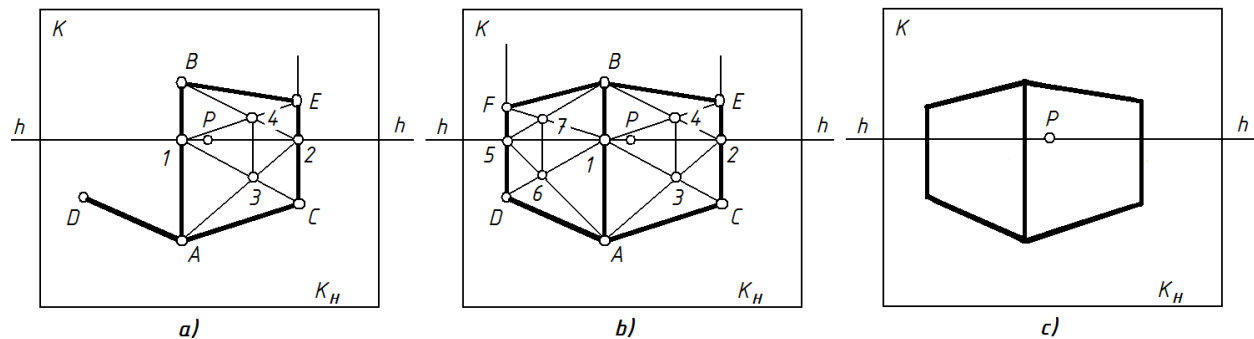


Figure 5

If the point B is below the horizon line, how is the line BE parallel to AC drawn? Logically, if point B is below the horizon line, then points 1 and 2 are determined by dividing the distances from points A and C to the horizon line, and 12 is the middle line of the rectangle formed up to the horizon line. The rest of the geometric constructions are done as above, and the line BE parallel to the line AC is drawn from the point B (Fig. 6).

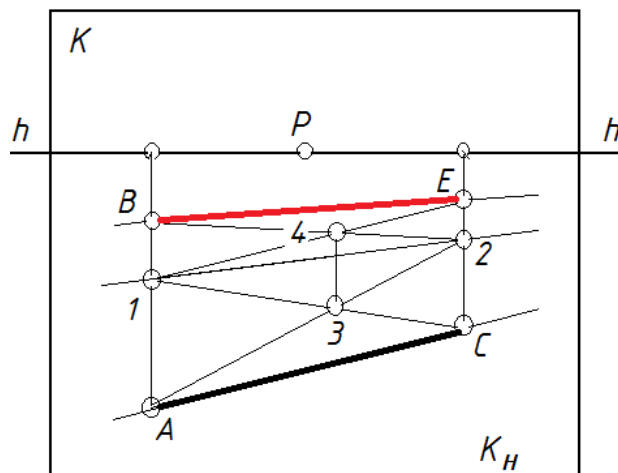


Figure 6

Example 3. Method of mutually parallel auxiliary horizontal straight lines. First of all, let's consider the scientific basis of this method. AB is vertical and AC is horizontal. Let it be required to draw a line BE parallel to AC from point B (Fig. 7).

To solve the problem, a horizontal straight line AC is drawn from point A with the point of descent outside

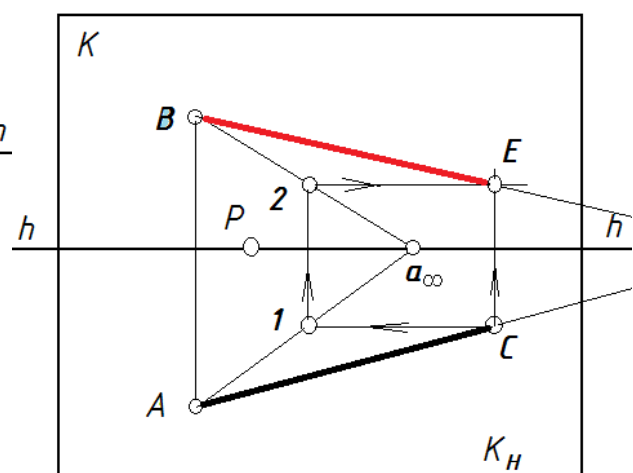


Figure 7

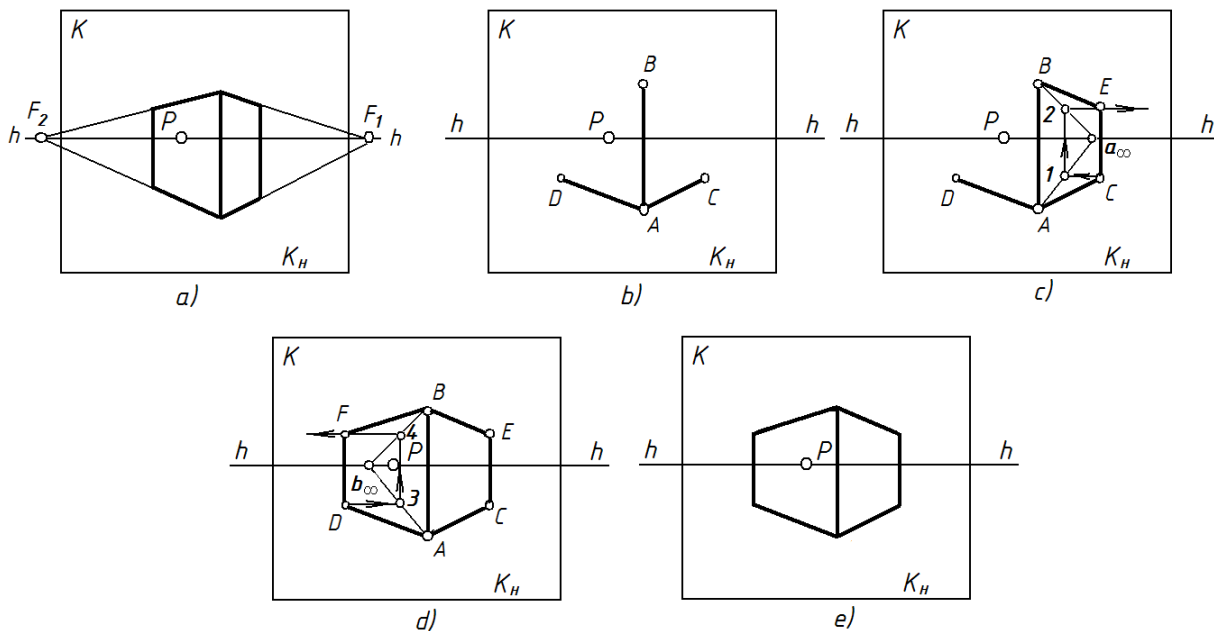
the border of the map. Then from points A and B, mutually parallel auxiliary horizontal straight lines are drawn, the point of descent of which is inside the border of the map  $a_{\infty}$ . A parallel line is drawn from point C to the horizon line, and point 1 intersecting it with  $Aa_{\infty}$  is determined. A vertical line drawn from point 1 intersects the section  $Ba_{\infty}$  at point 2, and a parallel line is drawn from this point to  $C_1$  (or the





horizon  $hh$  line). This line intersects with the vertical straight line drawn from point  $C$ , giving the desired point  $E$ . The points  $B$  and  $E$  are connected, and the line  $BE$  parallel to  $AC$  is divided, and the perspective of the surface  $ABEC$  is formed (Fig. 7).

It is possible to construct the perspective of a parallelepiped using the above method of mutually parallel auxiliary horizontal straight lines. The performed geometric construction is shown step by step and is understandable from the drawing (Fig. 8).



**Figure 8**

The method of mutually parallel auxiliary horizontal straight lines can also be applied to straight lines in the horizontal plane. It is well known from geometry that to draw a line  $A_4$  parallel to 12 given straight lines from point  $A$ , a straight line parallel to  $A_1$  is drawn from 2 points. By measuring the distance  $A_1$  to the left and right of this line, points 3 and 4 are determined. If point 1 is connected with point 3 and point  $A$  is connected with point 4, then  $13||A_2$  and  $A_4||12$  will be obtained and the problem will have a solution (Fig. 9, a).

The above rule can be used to draw a line  $A_4$  parallel to point  $A$  on the second border of the road width to the 12 edge border lines of the road given in the perspective. Of course, it is necessary to remember

and follow the rule of perspective, that is, parallel straight lines meet at a single point of descent (Fig. 9, b).

So, initially, the line  $A_1$  is taken parallel to the horizon line. Then points  $A$  and 2 are connected, and its falling point  $a_\infty$  is marked on the horizon line (of course,  $a_\infty$  does not have to go outside the border of the picture, if it does, you can choose another point in a convenient place of line 12 instead of point 2). Point 1 is also connected to  $a_\infty$ , and point 3 intersecting it with a line parallel to the horizon line from point 2 is determined. From point 2, a distance of 23 to the left is measured and 4 points are determined. Points  $A$  and 4 are



interconnected and the solution of the problem is reached (12||A4).

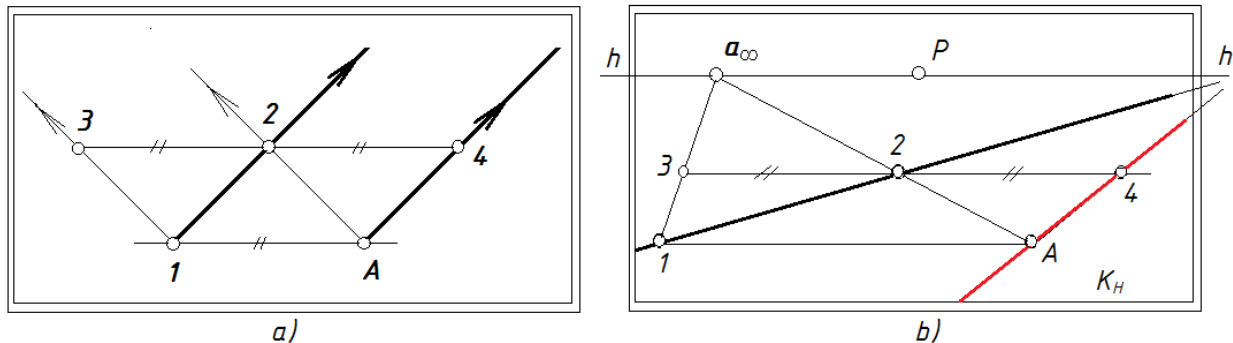


Figure 9

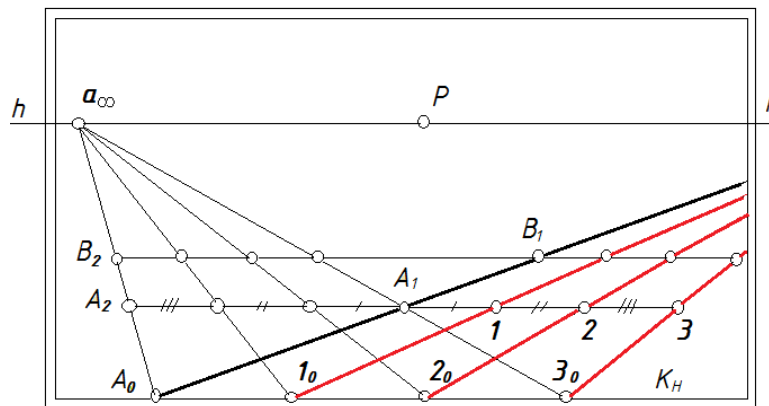


Figure 10

It is very convenient to construct a perspective of a set of mutually parallel straight lines using the method of auxiliary horizontal straight lines. The width of the auxiliary straight lines can be taken to any depth ( $A_1A_2$  or  $B_1B_2$ ) (Fig. 10).

Method of proportional sections. Cross-parallel straight lines without a drop point can be drawn in a vertical plane. The advantage of this method is that there are fewer geometric constructions performed on the picture plane, that is, fewer lines are drawn. Below is an example of this.

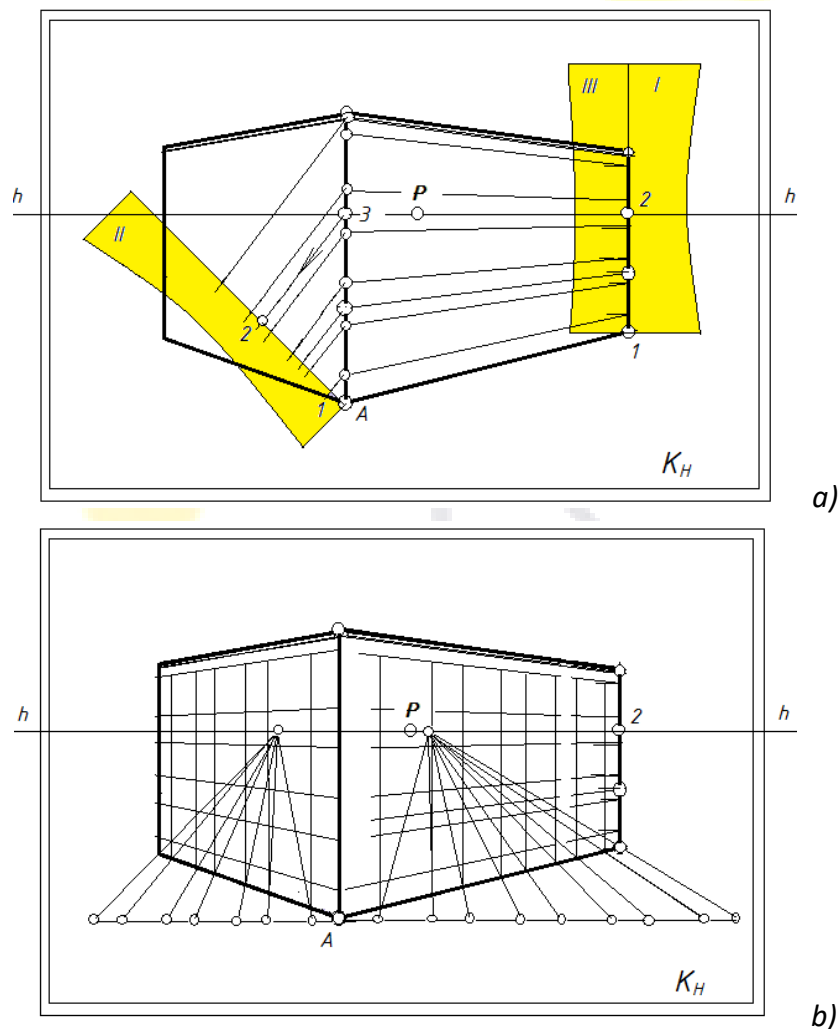
Figure 11, a shows the perspective of the two foundations, front corner (edge) of the building. The point of descent of horizontal straight lines in a mutually parallel situation is not given here. That is, there are floor and window boundaries at the front edge of the building, but the point of descent of the horizontal straight lines passing through these boundary points is not given on the surface of the image. A piece of paper is used to reduce the geometric constructions, and this piece of paper is placed on the right vertical edge of the building as in



case I. Points 1 at the base of the edge and 2 on the horizon line are assigned to it.

The piece of paper is then placed at an arbitrary angle to the leading edge as in case II. In this case, point 1 should overlap with point A, the base of the leading edge. Point 2 on the piece of paper and point 3 of the front edge intersecting with the horizon line are connected. From the positions (points) of the front edge of the building floors and window heights, parallel lines are drawn to 23 straight lines, and a number of points are determined on a piece of paper.

Then the piece of paper is placed on the right edge of the building again as in case III. The points identified on the piece of paper are marked on the right edge and are connected to the corresponding points on the leading edge. As a result, the eaves, floors and window heights of the building are divided (Fig. 11, a). Fales' theorem is applied here. In the same way, the eaves, floor and window heights on the left side of the building are determined.



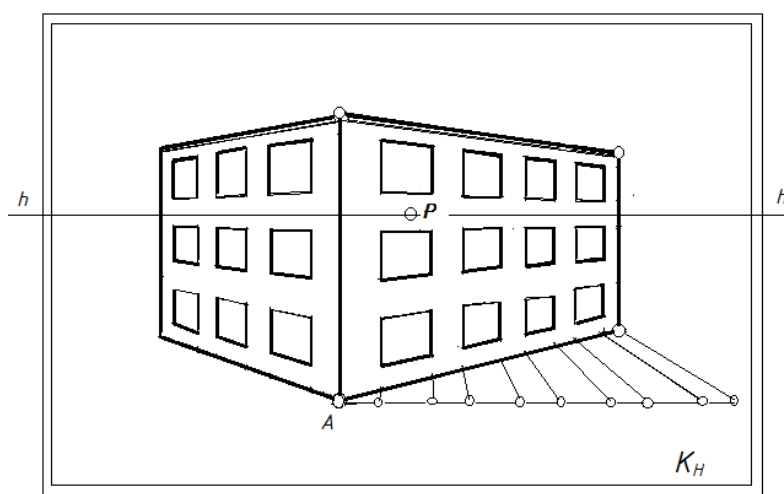


Figure 11

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To determine the width of the windows of the building, the perspective model (Fig. 11, b) of the rule of dividing the section into the given ratio is used (it is clear from the drawing). In Figure 11, d, we can see the completed perspective of the building.

## CONCLUSION

Various geometric constructions in orthogonal projection have been developed by scientists. In addition, a number of methods of checking the positional and metric relations between geometric shapes are known from educational literature. However, the favorable aspects of solving positional problems using geometric constructions in the central projection method (perspective) have been little studied. Positional issues in perspective can be widely used mainly in drawing. Taking this into account, the scientific-theoretical materials proposed in the article will be effective and useful if they are used not only in solving positional issues in perspective, but also in drawing. In addition, students will have the opportunity to learn the scientific basis of solving positional problems.



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