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A New Pedagogical Approach To Teaching The Theory Of Elementary Functions In School

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ABSTRACT

The article recommends teaching elements of mathematical analysis in school as the theory of elementary functions. It proposes changing the traditional order of topics "Limit of a sequence, limit of a function, derivative and integral" based on the properties of elementary functions to the following: "Limit of a sequence, limit, derivative and integral of a continuous function."

The simplification of the traditional approach was achieved by modifying the didactic axiom confirming the continuity of elementary functions and, consequently, altering the conditions in Heine's definition of function continuity. We believe that this pedagogical approach to teaching helps reduce theoretical gaps and associated complications in traditional teaching methods.

This work can be considered a simplified version of Academician A.N. Kolmogorov's idea that the theory of continuous functions should be taught in schools and the theory of general functions in higher education. In our opinion, basic and specialized schools can use the proposed project in developing their curricula.

Keywords: School mathematics, didactic approaches, continuous line and continuous function, didactic axiom, Heine's definition, methods of calculating limits, derivatives, and integrals,methodology of teaching elementary functions.

INTRODUCTION

Currently, it is a requirement of our time that every highly qualified specialist possess knowledge, skills, and abilities in the most essential branches of mathematics relevant to their field. Therefore, some concepts and principles of higher mathematics are now taught in the upper grades of school. In particular, elements of mathematical analysis topics such as limits, derivatives, and integrals are among these. The foundation for these topics is established using the concept of function limits.

In this article, we propose and substantiate a methodologically new approach to this issue by analyzing how elements of mathematical analysis are being addressed in schools of developed countries.

Based on the historical development of mathematics, we consider it appropriate to focus only on problems related to the theory of elementary functions (limits, derivatives, and integrals) in school, and to continue this study logically in higher education. To achieve this, we introduce a special definition of limits for elementary functions.

First, let's recall the methods used in schools to introduce and teach Cauchy and Heine definitions of sequence limits and function limits as described in the following literature[1 − 16]:

- Visualization (explanation using graphs) and interactive methods;
- Step-by-step method (from simple to complex);

- Using the concept of infinitesimal sequences or functions;
- Utilization of mathematical software (Maple, GeoGebra);
- Creative and interactive methods (group work, projects, practical exercises, and assignments);
- Application of pedagogical technologies (flipped classroom, problem-based learning), use of online platforms (Khan Academy, Coursera);
- Explaining the concept of approaching a certain point using real-life examples;
- Employing methods to develop students' logical thinking skills.

In this regard, some countries have adopted either the Cauchy or Heine definition of function limits as their primary approach, and have selected the following sequence of topics: Limits of sequences → Limits of functions → Derivatives → Integrals.

The methods presented in the aforementioned literature focus primarily on developing students' practical skills. However, due to the large volume and complexity of the material and limited time, the theoretical content is often presented without sufficient justification. In addition, not enough attention is paid to developing creative thinking. As a result, teaching function theory in higher education often has to start from scratch, and the expected outcomes may not be achieved.

In our opinion, considering the theoretical gaps in the current system and the limited number of teaching hours, it is possible to address these issues by implementing the study of elementary function theory in schools. We believe that teaching the general theory of functions in higher education based on the theory of elementary functions will enhance students' creative thinking abilities and produce higher quality future specialists.

Why should this approach be adopted? Looking at the history of mathematics, we see that the concept of a line predates the concepts of function and its graph. The graph of a function is merely one representation of a line. A continuous line is a fundamental concept, and the graphs of elementary functions consist of continuous lines. Taking this into account, we introduce the continuity of elementary function graphs in the results section of our article through a didactic axiom. Unlike the traditional method, this approach introduces the continuity of a function without relying on the concept of limits. Building on this, we then introduce the concept of limits as it pertains to elementary functions. Consequently, it becomes easier to justify many theoretical considerations.

METHODS

Our results depend on the methods described above for finding the limit of a sequence and the limit of a function using Heine's definition. According to Heine's definition, the function $f(x)$ has a finite limit L as $x \to a$ if the following holds: "For any sequence (x_n) converging to the number a, where $(x_n \neq a)$, if the sequence $f(x_n)$ converges to some number L, then this number is called the limit of $f(x)$ as $x \to a$, and is written as: $\lim_{x \to a} f(x) = L$".

In the special case $L = f(a)$, the function is called continuous at the point $x = a$.

In the definitions of Heine or Cauchy, the continuity of a function at a point or on a set is expressed by the concept of a limit. If we consider the graph of elementary functions as a continuous line, then continuity can be introduced as an intuitive initial concept using the axiomatic method. In other words, we can claim the continuity of elementary functions in the domain of definition without the concept of a limit, and we do not contradict this.

Let the graph of the function be a continuous line. Let us represent on the graph of the function some sequence (x_n) and the corresponding sequence $\left(f(x_n)\right)$, which tends to a. In this case, we obtain points (n, x_n)corresponding to $(n; f(x_n))$. In $n \to \infty$, these points tend to the point $(\infty; f(a))$ on the graph of the function, that is, the relation $\lim_{n \to \infty} f(x_n) = f(a)$ holds. Similarly, any sequence (y_n): $y_n \to a$ $n \to \infty$ different from (x_n) and the corresponding sequence of points on the graph $(n; f(y_n))$ in $n \to \infty$ will also be $\left(\infty, f(a)\right)$, that is $\lim_{n \to \infty} f(y_n) = f(a)$ will be true. From this, the following conclusion can be drawn. When $f(x)$ is continuous, the expression "for any sequence (x_n) $(x_n \neq a)$" in Heine's definition of function continuity at $x = a$ can be replaced with "for some sequence (x_n) with $(x_n \neq a)$, that tends to a" that tends to a.

RESULTS

The object of our research is the class of elementary functions (polynomials, trigonometric, exponential, logarithmic functions), and our aim is to describe their differential and integral calculus.

It is known that a line is one of the fundamental concepts in geometry. The graph of a function is a special type of line, the points of which are determined by a specific formula. A continuous function is a line whose graph can be drawn over a certain interval without lifting the pen, i.e., it is a line without breaks or discontinuities.

Based on the above, we present our didactic axiom that affirms the continuity of elementary functions without proof.

Axiom (didactic). **Elementary functions are represented as continuous lines in their domain.**

$x^2, sinx,\ lnx,\ e^x$ For example, the graphs of elementary functions such as are represented as continuous lines in their domain.

Now let's proceed to the definition of the limit of an elementary function.

Let $f\ (x)$ be an elementary function, $D\ (f)$ be its domain, and $a\epsilon R$ be some real number. $a \in D(f)$ and $f(a)$ be the values of the function at $x = a$.

Definition 1: **If for some sequence (x_n) $(x_n \neq a)$, approaching a, the sequence $(f(x_n))$ approaches the number $f(a)$, this number is called the limit of $f(x)$ as $x \to a$ and is written as $\lim_{x\to a} f(x) = f(a)$**

$\lim_{x\to 2} x^2$ For example, suppose we need to determine. By definition

$x_n = 2 + \frac{1}{n} \to 2,\ n \to \infty$, we take a sequence and

$f(x_n) = (2 + \frac{1}{n})^2 = 4 + \frac{2}{n} + \frac{1}{n^2}$, $n = 1,2\ldots$, We determine the limit of the sequence $f(x_n)$.

$\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty}(4 + \frac{2}{n} + \frac{1}{n^2}) = 4 + 0 + 0 = 4$ $\lim_{x\to 2} x^2$ For example, suppose we need to determine. By definition

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$\lim_{x\to 2} x^2 = 4$

$\lim_{x\to 1} arcsinx = ?$ $x_n = 1 - \frac{1}{n} \to 1,\ n \to \infty$ $\lim_{n\to\infty} f(x_n) = \lim_{n\to\infty} arcsin(1 - \frac{1}{n}) = arcsin1 = \frac{\pi}{2}$ For example, There fore. $\lim_{x\to 1} arcsinx = \frac{\pi}{2}$

Note According to the traditional Heine definition, our conclusion in this case would be incorrect, because we did not consider an arbitrary sequence.

If $a \notin D(f)$ and $L \in R$ is some finite number, then the limit of the function on $x \to a$ is defined as follows.

Definition 2 : **If for some sequence (x_n) approaching a, the sequence $(f(x_n))$ tends to a number L, then this number is called the limit of $f(x)$ as $x \to a$, and it is written as $\lim_{x\to a} f(x) = L$.**

Similar definitions can be formulated for the cases when $= \pm\infty$, and when L is either finite or infinite. Now, based on the axioms and definitions, we will derive the definition of the derivative of a function at a point x_0.

Definition 3 : **If there exists a limit $\lim_{n\to\infty} \frac{f(x_n)-f(x_0)}{x_n - x_0}$ for some sequence $(x_n)(x_n \neq a)$, approaching x_0 , then it is called the derivative of the function $f(x)$ at the point $x = x_0$ and is denoted as $f\,'(x_0)$.**

For example, let f(x) $= x^2$ and suppose we are asked to find $f\,'(x_0)$. According to the definition, we take $x_n = x_0 + \frac{1}{n}$, with $n \to \infty$, and compute the following limit: $\lim_{n\to\infty} \frac{f(x_n)-f(x_0)}{x_n - x_0}$

$$\lim_{n\to\infty} \frac{(x_0 + \frac{1}{n})^2 - {x_0}^2}{x_0 + \frac{1}{n} - x_0}$$

$$= \lim_{n\to\infty} \frac{\frac{2x_0}{n} + \frac{1}{n^2}}{\frac{1}{n}} = \lim_{n\to\infty}(2x_0 + \frac{1}{n}) = 2x_0$$

So it's $f\,'(x_0)$ in this case.

Thus, at $x = x_0$, we have $(x^2)' = 2x_0$

A new pedagogical approach to teaching the theory of elementary functions at the school level is implemented through solving the following problems based on established axioms and definitions:

- *Constructing the table of derivatives of elementary functions, applying the rules of differentiation, studying the fundamental theorems related to differentiable functions, and determining the intervals of monotonicity and extrema of functions;*
- *Solving problems involving the indefinite integrals of elementary functions, evaluating definite integrals, and applying the Newton-Leibniz formula.*

Certainly, solving such problems is relatively easier compared to the general theoretical framework. If students pursue higher education later on, it will not be difficult to explain these concepts and theorems to them in a generalized and comprehensive manner.

DISCUSSION

The purpose of the study was to substantiate a new pedagogical approach to teaching differential and integral calculus of the theory of elementary functions in school. To achieve this goal, we proposed two theoretical tools in the results section.

The first is a didactic axiom corresponding to elementary functions, in which we accepted without proof that elementary functions are represented as continuous lines within their domain of definition. This didactic axiom allows school students to consider elementary functions as continuous functions.

In the theory of functions section of higher mathematics, the proof of this introduced axiom is presented as a theorem. Therefore, we can justifiably claim that the didactic axiom we introduced is consistent. Some schools worldwide have organized their educational process using such didactic axioms. However, the axiom we propose is related to the concept of a continuous line in accordance with the history of mathematics. Furthermore, the study of function theory originally began with elementary functions.

With the acceptance of the axiom, a second theoretical instrument emerges. Now, if we present the traditional Heine definition of the limit of a function for a continuous line, i.e., a continuous function, we derive the definition of the limit of elementary functions in terms of sequences. In definitions 1 and 2 in the results section, we now see that the phrase "for any sequence (x_n) approaching a " in Heine's definition has been replaced by "for some sequence (x_n) approaching a." "If students pursue higher education, encountering Heine's general definition will not come as a surprise to them". In particular, they can understand the derivation of definitions 1 and 2. The scheme for studying function theory in higher education is as follows: "Sequence limit $\rightarrow$ function limit $\rightarrow$ function continuity $\rightarrow$ function derivative $\rightarrow$ function integral," while in elementary function theory, this scheme becomes: "Sequence limit $\rightarrow$ continuous function limit $\rightarrow$ function derivative $\rightarrow$ function integral." These topics are within the scope of elementary functions, briefly stating theoretical and practical results only for elementary functions. In higher education, issues of generalization are considered. This, of course, helps to enhance the student's creative abilities.

Now let's move on to recommendations for the practical application of our research results. It is known that schools worldwide are often of two types. Basic schools - where students learn concepts necessary for life. The second is specialized schools - where students study concepts necessary for preparing for higher education in the future. In Uzbekistan, these schools are called "Presidential Schools" and "Academic Lyceums."

"The new pedagogical approach proposed in this article is designed for specialized schools. However, it can be simplified and adapted for use in general education schools as well. In our future research, we plan to develop a teaching manual".

The idea we introduced is not new. The concept of teaching only the theory of continuous functions in school and the theory of general functions in higher education was proposed by Academician A.N. Kolmogorov. An attempt to implement this idea in a strong form was made by Y.N. Vilenkin and A.G. Martkovich in the literature [11]. In it, the teaching of elementary functions in the form of "Sequence limit $\rightarrow$ function limit $\rightarrow$ function continuity $\rightarrow$ function derivative $\rightarrow$ function integral" remains

The continuity of elementary functions is proven using Cauchy's definition, and the main instrument is defined as the limit of a continuous function: $\lim_{x\to a} f(x) = f(a)$. Of course, teaching in this method is more rigorous than our approach, but it requires strong schools for practical application.

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