

RESEARCH ARTICLE OPEN ACCESS

A Framework for Enhancing Students' Mathematical Proficiency Through Meaningful Instructional Design Strategies

Dr. Nadeesha Perera

Department of Linguistics, Sri Lanka

Dr. Tharindu Jayawardena

Department of Language and Literary Studies, Sri Lanka

Received: 16 August 2026 Accepted: 10 September 2026 Published: 12 October 2026

ABSTRACT

Mathematical proficiency requires more than procedural accuracy; it involves conceptual understanding, mathematical reasoning, problem-solving, communication, and the ability to apply mathematical knowledge in unfamiliar situations. Conventional instructional practices that emphasize memorization may not adequately support these interconnected competencies. This paper develops a conceptual framework for enhancing students' mathematical proficiency through Meaningful Instructional Design (MID), emphasizing the integration of prior knowledge, contextual learning, active engagement, mathematical representation, collaborative inquiry, and reflective assessment. A reference-based analytical review was adopted to synthesize the eleven supplied reference entries concerning meaningful instructional design, mathematical literacy, problem-solving, reasoning, communication, and learning outcomes. The proposed framework organizes instruction into six connected phases: diagnostic assessment, contextual activation, meaningful knowledge construction, guided mathematical application, collaborative reasoning, and reflective evaluation. The analysis indicates that meaningful instructional design provides a coherent basis for aligning instructional activities with multiple dimensions of mathematical proficiency. Its potential effectiveness, however, depends on appropriate scaffolding, the quality of mathematical tasks, assessment alignment, and responsiveness to individual learning differences. The framework offers a structured approach for instructional planning and future empirical evaluation. Because the present study is conceptual rather than an original classroom experiment, its proposed outcomes require validation through systematic implementation and comparative research.

Keywords: Meaningful Instructional Design, Mathematical Proficiency, Mathematics Education, Conceptual Understanding, Mathematical Reasoning, Problem-Solving Skills, Mathematical Literacy, Formative Assessment, Contextual Learning.

1. INTRODUCTION

Mathematical proficiency is a multidimensional educational outcome encompassing the understanding of mathematical concepts, the selection and execution of appropriate procedures, logical reasoning, problem-

solving, and the communication of mathematical ideas. Students may successfully reproduce familiar algorithms yet struggle to interpret unfamiliar problems or explain why a particular procedure is appropriate. This difference highlights the limitations of evaluating mathematical

learning exclusively through procedural performance.

Instructional design is therefore important because it determines how mathematical concepts are introduced, connected to existing knowledge, practiced, discussed, and assessed. Meaningful Instructional Design (MID) provides a basis for organizing learning experiences around the construction of understanding rather than the passive reception of information. Anggraini (2023) examines the influence of the MID learning model on students' motivation and cognitive learning outcomes, establishing a relevant direction for considering the relationship between instructional organization and learning performance. However, the implications for broader mathematical proficiency require an integrated framework that distinguishes immediate cognitive outcomes from the development of transferable mathematical competencies.

The relevance of this approach is also reflected in research addressing mathematical problem-solving, literacy, reasoning, and communication. Apriani et al. (2023a) investigate MID in relation to mathematical problem-solving involving systems of three-variable linear equations, while Glean et al. (2023) examine mathematical reasoning in the context of arithmetic sequences and series. Arifani et al. (2021) connect mathematical literacy with self-directed learning and outdoor learning based on MID. Together, these research directions suggest that meaningful learning should be evaluated through several dimensions rather than a single measure of achievement.

Nevertheless, an instructional model does not automatically produce comprehensive learning. Its educational value depends on how teachers select tasks, activate prior knowledge, provide feedback, support different learning needs, and determine whether students can transfer knowledge beyond the original instructional context. A framework must therefore explain not only the stages of instruction but also the mechanisms connecting those stages to measurable mathematical competencies.

The objective of this paper is to develop a theoretically organized framework for enhancing mathematical proficiency through meaningful instructional design strategies. Specifically, it synthesizes the supplied literature, identifies relevant instructional mechanisms, proposes a structured implementation model, and establishes criteria for evaluating its potential effectiveness. The study is limited to conceptual analysis of the supplied references and does not claim

experimentally verified improvements in student performance.

2. LITERATURE REVIEW

2.1 Meaningful Instructional Design and Learning Outcomes

Meaningful Instructional Design can be understood as an instructional approach that organizes learning so that new information is connected to learners' existing knowledge, experiences, and cognitive structures. Rather than treating mathematical procedures as isolated rules, the approach encourages students to understand relationships among concepts, interpret mathematical representations, and apply acquired knowledge in relevant situations.

Anggraini (2023) focuses on the relationship between MID, student motivation, and cognitive learning outcomes. This research direction is important because motivation can influence students' willingness to engage with challenging mathematical tasks, while cognitive outcomes indicate whether instructional activities support learning. Nevertheless, motivation and achievement should be distinguished analytically: increased engagement does not necessarily demonstrate conceptual mastery, and improved performance on familiar exercises does not independently establish transfer to unfamiliar problems.

Kusumawati et al. (2024) examine the implementation of a Meaningful Instruction Design model assisted by comics in students' understanding of multiplication. This topic illustrates how representational resources can make mathematical relationships more accessible. Visual narratives may provide concrete contexts for abstract operations, although their effectiveness depends on whether the representations clarify mathematical structure rather than merely decorate instructional materials.

Gazali and Atsnan (2022) address contextual approaches to meaningful mathematics learning. Contextualization can help students recognize the relationship between mathematics and everyday situations. For example, percentages can be introduced through price reductions, while ratios can be explored through recipes or scale drawings. However, contextual instruction must eventually connect the situation to generalized mathematical principles; otherwise, students may learn to solve only the specific contextual problem presented.

2.2 Mathematical Problem-Solving and Reasoning

Mathematical problem-solving requires students to interpret a question, identify relevant information, select an approach, perform calculations, and evaluate the plausibility of a result. Apriani et al. (2023a) and Apriani et al. (2023b) address the application of MID to mathematical problem-solving in systems of three-variable linear equations. Their shared subject provides a relevant basis for considering how structured instruction can support multistep mathematical tasks.

Such tasks require more than computational fluency. Students must translate verbal information into equations, recognize relationships among variables, select a solution method, and interpret the resulting values. Meaningful instruction can support these processes by connecting symbolic representations to concrete situations and by requiring students to explain why their chosen procedures are appropriate.

Glean et al. (2023) focus on mathematical reasoning associated with arithmetic sequences and series. Reasoning in this domain involves identifying patterns, expressing general rules, justifying relationships, and distinguishing observed regularities from mathematically supported conclusions. An effective instructional design should therefore provide opportunities for students to predict, test, explain, and generalize rather than simply memorize formulas.

Arifani et al. (2021) connect mathematical literacy with self-directed learning in an outdoor learning context based on MID. This research direction broadens the instructional perspective beyond classroom-based exercises. Real-world environments can provide opportunities to measure quantities, compare relationships, estimate values, and interpret numerical information. Nevertheless, practical activities require explicit mathematical guidance to ensure that observation develops into rigorous mathematical interpretation.

2.3 Mathematical Communication, Representation, and Learner Differences

Mathematical proficiency also depends on students' ability to express mathematical ideas through words, symbols, diagrams, equations, and logical arguments. Al-Hanifah et al. (2023) examine analytical thinking and mathematical communication in relation to an online group investigation

learning model. Although the instructional model differs from MID, the study provides a relevant complementary perspective on collaborative inquiry and communication as dimensions of mathematical learning.

Manik et al. (2020) investigate students' mathematical communication through problem-based learning using a quasi-experimental approach. This focus reinforces the importance of examining not only the answers students produce but also how they represent and explain mathematical relationships. For instance, students solving a proportional reasoning task might be asked to present a numerical solution, draw a diagram, and explain why the quantities maintain a constant ratio.

Individual learning differences are also relevant. Mulyono et al. (2018) examine self-assessment in mathematical proportional application within MID learning in relation to students' learning styles. This perspective suggests the importance of examining how learners monitor their progress and how instructional support interacts with individual differences. However, learner categories should not be treated as fixed determinants of achievement; instructional decisions should instead be guided by demonstrated understanding, task performance, and students' changing learning needs.

Karim (2022) presents a bibliometric analysis of trends in applied mathematics research using VOSviewer and Google Scholar. Although this work does not directly establish the effectiveness of MID, it represents a methodological perspective on mapping research trends. It also highlights the distinction between identifying patterns in scholarly publications and establishing causal evidence about classroom interventions.

2.4 Research Gap and Theoretical Positioning

The supplied literature addresses several complementary dimensions of mathematics education: motivation and cognitive outcomes, contextual learning, problem-solving, reasoning, mathematical literacy, communication, visual representations, and self-assessment. However, the reference set does not itself establish a single empirically validated framework integrating all these dimensions into one instructional sequence.

The central research gap addressed here is therefore organizational and evaluative. Rather than assuming that one instructional technique independently develops every

mathematical competency, the proposed framework specifies how different strategies can be coordinated and how their contributions can be assessed.

The framework positions meaningful instructional design as an organizing structure, contextual tasks as the medium for application, collaborative activities as opportunities for explanation, and formative assessment as a mechanism for monitoring learning. Anggraini (2023) is particularly relevant to this positioning because motivation and cognitive outcomes represent complementary indicators that can be incorporated into a broader evaluation system without being treated as interchangeable measures of proficiency.

3. METHODOLOGY

3.1 Research Design

This study employs a conceptual, reference-based analytical review to develop a framework for enhancing mathematical proficiency through meaningful instructional design. It does not involve the recruitment of students, the administration of an intervention, or the collection of original classroom performance data. Its primary purpose is to organize the concepts represented in the supplied references into an instructional model that can subsequently be tested empirically.

The analysis follows three broad operations: identification of recurring educational constructs, synthesis of their instructional relationships, and formulation of a staged framework with corresponding evaluation criteria. These operations are intended to make the reasoning behind the proposed model explicit rather than to imply that a formal systematic review or statistical meta-analysis was conducted.

3.2 Source Selection and Analytical Scope

The analytical corpus consists of the eleven reference entries provided by the user. These entries address meaningful instructional design, mathematics learning outcomes, contextual approaches, problem-solving, mathematical reasoning, mathematical literacy, communication, self-assessment, and research mapping.

The references were grouped thematically according to their stated titles and bibliographic information. Studies addressing MID were used to establish the framework's

central instructional orientation. Studies addressing communication, problem-based learning, and group investigation were used to identify complementary strategies. Research concerning outdoor learning and comic-assisted instruction informed the discussion of contextual and visual representations.

This process is a conceptual synthesis rather than a complete appraisal of the underlying articles. Because the full texts, research instruments, sample characteristics, and statistical results were not supplied, the analysis does not infer effect sizes, sample-specific outcomes, or causal relationships that cannot be verified from the available information.

3.3 Operational Definition of Mathematical Proficiency

For the purposes of this framework, mathematical proficiency is operationalized through five interrelated dimensions.

Conceptual understanding refers to recognizing mathematical relationships and explaining why procedures work. Procedural fluency concerns accurate and efficient execution of appropriate methods. Strategic problem-solving involves interpreting unfamiliar tasks, selecting suitable representations, and developing solutions. Mathematical reasoning and communication encompass the construction, justification, and explanation of mathematical arguments. Transfer and self-regulation refer to applying knowledge in new contexts and evaluating one's own understanding.

These dimensions are analytical categories developed for the present framework, not a claim that every supplied study used the same assessment structure. They allow instructional strategies to be connected to specific learning outcomes rather than evaluated solely through a general achievement score.

3.4 Proposed Meaningful Instructional Design Framework

The proposed framework contains six sequential but iterative phases. Each phase has an instructional purpose, a teacher action, a student activity, and an observable form of evidence.

Phase 1 — Diagnostic Assessment

Identify prior knowledge, misconceptions, and readiness.

Phase 2 — Contextual Activation

Connect the mathematical topic to meaningful problems and familiar experiences.

Phase 3 — Knowledge Construction

Develop concepts through examples, diagrams, symbolic representations, and guided inquiry.

Phase 4 — Guided Application

Apply concepts to increasingly complex mathematical tasks.

Phase 5 — Collaborative Reasoning

Compare strategies, justify conclusions, and communicate mathematical thinking.

Phase 6 — Reflection and Evaluation

Assess mastery, review errors, and transfer learning to unfamiliar tasks.

Figure 1. Proposed instructional sequence developed for this conceptual study. Assessment evidence informs the next instructional cycle.

Phase 1: Diagnostic assessment. Teachers identify students' existing knowledge through short problems, oral questions, written explanations, or concept maps. Before introducing linear equations, for example, a teacher may assess whether students understand variables, equality, and basic algebraic operations. This phase prevents the instructional sequence from assuming prerequisite knowledge that students may not possess.

Phase 2: Contextual activation. Teachers introduce a situation that creates a meaningful reason to learn the target concept. A problem involving the combined costs of different items may introduce systems of equations. Contexts should be accessible without oversimplifying the mathematics. The goal is to make the underlying relationship recognizable and then translate it into a formal mathematical representation.

Phase 3: Knowledge construction. Students connect contextual information to diagrams, tables, equations, and generalized principles. Teachers provide worked examples and progressively reduce support as students demonstrate understanding. Comic-assisted representations, as considered by Kusumawati et al. (2024), may be useful where visual sequencing clarifies a mathematical operation. The critical criterion is whether students can explain the mathematical meaning represented.

Phase 4: Guided application. Students solve tasks of increasing complexity while receiving feedback on their reasoning and procedures. In a lesson on arithmetic sequences, learners might identify a common difference, generate further terms, and derive a general expression. Tasks should move from familiar examples toward variations that require students to decide which procedure is appropriate.

Phase 5: Collaborative reasoning. Students compare solution methods, explain assumptions, and examine errors. Group investigation and problem-based activities can support mathematical communication, as suggested by the research directions of Al-Hanifah et al. (2023) and Manik et al. (2020). Collaboration must include individual accountability so that participation does not conceal gaps in understanding.

Phase 6: Reflection and evaluation. Students complete individual assessment tasks, explain the strategies they used, and identify unresolved difficulties. Teachers compare current performance with the diagnostic baseline and use the evidence to determine subsequent instructional support. This phase reflects the importance of motivation, cognitive outcomes, and self-assessment discussed in Anggraini (2023) and Mulyono et al. (2018).

3.5 Instructional Alignment and Assessment

The six phases should be aligned with explicit learning objectives. If the objective is to develop mathematical reasoning, an assessment that records only final numerical answers is insufficient. Students should also explain their method, justify intermediate steps, or compare alternative solutions. If the objective is transfer, the assessment should include a problem that differs meaningfully from the examples practiced during instruction.

A proposed assessment matrix is presented below.

Proficiency dimension	Instructional evidence	Suggested evaluation
Conceptual understanding	Explanations and representations	Conceptual questions
Procedural fluency	Accuracy and consistency	Structured exercises
Problem-solving	Strategy selection and solution process	Multistep problems
Reasoning and communication	Justifications and mathematical dialogue	Explanation rubric
Transfer and self-regulation	Performance on unfamiliar tasks and reflection	Transfer task and self-assessment

The matrix is a proposed design instrument, not a validated scale. Its reliability and validity would need to be investigated before use in formal comparative research.

3.6 Illustrative Implementation Scenario

Consider a hypothetical secondary-school lesson on systems of three-variable linear equations. The learning objective is for students to formulate equations from a practical situation, solve the system, explain the solution, and evaluate its meaning.

The teacher begins with a diagnostic task involving two variables and checks whether students understand equations and substitution. Students then receive a contextual problem involving three products with different prices and several combinations of purchases. They represent the information using a table, identify unknown quantities, and formulate three simultaneous equations.

During knowledge construction, the teacher demonstrates how algebraic representations correspond to the contextual relationships. Students then solve a related problem independently and in small groups. Group members compare elimination and substitution strategies, identify calculation errors, and explain why a proposed solution satisfies all three equations. The final assessment introduces a new purchasing situation to determine whether students can transfer their understanding.

This scenario illustrates how contextualization, procedural practice, collaboration, and reflection can be coordinated. It does not constitute evidence that the framework has already improved student outcomes. Its value lies in specifying instructional actions and observable learning indicators that can be evaluated through a future classroom study.

3.7 Proposed Evaluation Design

The framework could be evaluated through a quasi-experimental study involving comparable student groups. One group would receive instruction organized around the proposed MID framework, while the comparison group would receive the school's existing instructional approach. Both groups would complete equivalent pre-intervention and post-intervention assessments covering conceptual understanding, procedural fluency, problem-solving, reasoning, and transfer.

The analysis should compare changes in outcomes while accounting for baseline performance and relevant classroom differences. Student motivation could be measured separately using an appropriately validated instrument, since motivational responses should not be substituted for evidence of mathematical mastery. Classroom observations, student explanations, and teacher interviews could provide additional information about implementation quality.

Such an evaluation would require appropriate sampling, informed consent where applicable, consistent assessment procedures, and transparent reporting of missing data and limitations. Until these steps are completed, the framework should be treated as a testable instructional proposal rather than a demonstrated intervention with established effect sizes.

4. RESULTS

4.1 Integrated Dimensions of Mathematical Proficiency

The conceptual synthesis identifies five dimensions that can be coordinated through meaningful instructional design: conceptual understanding, procedural fluency,

strategic problem-solving, mathematical reasoning and communication, and transfer with self-regulation. These dimensions provide a more informative analytical structure than treating mathematical proficiency as a single undifferentiated outcome.

The first finding is that mathematical proficiency requires alignment between instructional activity and the intended learning outcome. For example, practicing equations can support procedural fluency, but the same activity may provide insufficient evidence of whether students understand the relationships represented by those equations. The problem-solving focus of Apriani et al. (2023a) and Apriani et al. (2023b), together with the reasoning focus of Glean et al. (2023), supports the relevance of evaluating the processes through which students reach mathematical conclusions.

The framework consequently recommends that teachers assess both the correctness of an answer and the reasoning that produces it. A student who arrives at the correct result through an invalid argument may need a different form of instructional support from a student who understands the concept but makes an arithmetic error. Distinguishing these cases enables more precise instructional decisions.

4.2 The Role of Contextual and Representational Learning

The second finding concerns the instructional function of context and representation. Contextual problems can make mathematical relationships easier to recognize by giving students a reason to formulate equations, compare quantities, or interpret numerical patterns. Gazali and Atsnan (2022) address contextual approaches to meaningful mathematics learning, while Arifani et al. (2021) examine mathematical literacy in relation to self-directed and outdoor learning within an MID context.

These research directions suggest that mathematical learning can be connected to experiences beyond routine textbook exercises. However, the conceptual analysis also identifies a design requirement: contextual activity must lead toward mathematical abstraction. A lesson involving shopping, for example, should not end with the calculation of a particular price. Students should also identify the general principles governing percentages, ratios, or simultaneous relationships.

Representational resources serve a related function. Visual

narratives and diagrams can make relationships more accessible, as illustrated by the topic examined by Kusumawati et al. (2024). Their instructional value depends on whether students can connect visual information with symbolic expressions and explain the relationship between them. A representation that attracts attention without clarifying the mathematical concept may contribute little to learning.

Accordingly, the framework treats context and representation as bridges between experience and abstraction, rather than as independent indicators of instructional success.

4.3 Motivation, Engagement, and Cognitive Outcomes

The third finding is that motivational indicators and cognitive outcomes should be evaluated jointly but separately. Anggraini (2023) focuses on motivation and cognitive learning outcomes within meaningful instructional design. This focus is relevant because students who recognize the purpose of a mathematical task may be more willing to participate, persist through difficulty, and engage with feedback.

Nevertheless, motivation is not equivalent to mathematical competence. Students can enjoy collaborative activities while retaining misconceptions, and students can acquire procedural knowledge without expressing strong interest in the subject. A comprehensive evaluation should therefore examine engagement alongside independent evidence of understanding, reasoning, and transfer.

The proposed framework incorporates this distinction by placing motivational observations within the instructional process and mathematical performance within the outcome assessment. Participation, task completion, and students' reported confidence may help teachers interpret learning conditions, but they cannot independently establish improvement in proficiency.

The analytical implication is that meaningful instruction should be designed to support both engagement and intellectual rigor. Activities should be accessible enough to encourage participation while demanding sufficient explanation and reasoning to reveal whether genuine learning has occurred.

4.4 Collaboration, Communication, and Self-Regulation

The fourth finding concerns the relationship between collaborative learning and individual mathematical understanding. Al-Hanifah et al. (2023) examine analytical thinking and mathematical communication in online group investigation, while Manik et al. (2020) address mathematical communication in problem-based learning. These topics indicate the relevance of studying how students formulate and communicate mathematical ideas within interactive learning environments.

Collaborative discussion can reveal alternative approaches, expose incorrect assumptions, and encourage students to justify their conclusions. Yet group performance can obscure individual difficulties when one student completes most of the mathematical work. The framework therefore combines group problem-solving with independent tasks and individual explanations.

Self-regulation provides a further link between learning activities and sustained competence. Mulyono et al. (2018) examine self-assessment in mathematical proportional application within MID learning. In the proposed framework, students use reflection to identify errors, judge the adequacy of their strategies, and determine what they need to practice next. Such reflection is most useful when supported by explicit criteria and teacher feedback rather than unsupported judgments of ability.

4.5 Framework-Level Findings and Evidence Boundaries

The fifth finding is that the principal contribution of the proposed framework is the organization of complementary instructional strategies into a coherent sequence. Diagnostic assessment informs contextual activation; contextual activation supports knowledge construction; guided application develops fluency; collaboration makes reasoning visible; and reflection informs subsequent instruction. Assessment operates across all phases rather than appearing only at the end.

Karim (2022) contributes a methodological reminder that bibliometric mapping and analysis of research trends are distinct from evaluating intervention effectiveness. In the same way, conceptual agreement among the topics represented in the supplied literature should not be mistaken for a quantitative estimate of the effectiveness of the proposed framework.

The findings are therefore analytical propositions derived

from thematic synthesis, not experimentally measured outcomes. No numerical gains, statistical significance, or comparative effect sizes can be reported from the supplied bibliographic entries alone. The appropriate conclusion is that the framework offers a coherent structure for instructional planning and empirical investigation, while its actual effects remain to be established.

5. DISCUSSION

5.1 Theoretical Implications of the Proposed Framework

The proposed framework contributes to mathematics education by treating meaningful instructional design as an organizing principle for coordinating multiple learning processes. Instead of assuming that one instructional activity can independently improve all dimensions of proficiency, it establishes relationships among prior knowledge, contextual interpretation, conceptual construction, practice, communication, and reflection.

This organization is important because mathematical learning involves transitions between different forms of understanding. Students may recognize a pattern visually without being able to express it algebraically, or they may perform a familiar procedure without recognizing when it applies. The framework addresses these potential discontinuities by requiring students to move between contextual, visual, verbal, and symbolic representations.

The literature topics concerning mathematical problem-solving, reasoning, and literacy reinforce the need for this broader perspective (Apriani et al., 2023a; Arifani et al., 2021; Glean et al., 2023). The contribution of the present framework is not to claim that these studies validate one common instructional sequence, but to use their complementary research directions to identify instructional functions that can be deliberately coordinated.

A further theoretical implication concerns the distinction between instructional activities and learning outcomes. Collaborative work, contextual tasks, and visual resources are means of supporting learning, not proof that learning has occurred. Their effectiveness must be assessed against explicit objectives and evidence of individual understanding. This distinction helps prevent the mistaken assumption that an engaging classroom automatically produces deep mathematical proficiency.

5.2 Implications for Classroom Practice

For teachers, the framework offers a practical basis for designing lessons around identifiable learning difficulties. A teacher introducing fractions, for instance, might begin by assessing students' understanding of equal parts, connect fractions to measurement or sharing, represent relationships visually, and gradually introduce equivalent fractions and symbolic operations. Students would then solve problems, explain their strategies, and complete an independent assessment involving a different context.

This sequence can be adapted to different mathematical topics, but adaptation should preserve the underlying instructional logic. Teachers should not introduce elaborate contexts where a simple representation would be more effective, nor should they remove contextual support before students have established sufficient conceptual understanding.

Anggraini (2023) provides a relevant research direction for considering motivation alongside cognitive outcomes. In classroom practice, teachers can use clear objectives, appropriately challenging problems, timely feedback, and visible progress indicators to support engagement. However, motivational strategies should not lower expectations for mathematical reasoning. Students should still be required to explain methods, identify errors, and demonstrate independent mastery.

Assessment should also become more diagnostic. Rather than marking an answer simply correct or incorrect, teachers can identify whether a difficulty arises from conceptual misunderstanding, an inappropriate strategy, a procedural error, or weak interpretation of the problem. Each error type calls for a different response. Conceptual misunderstandings may require alternative representations, whereas procedural errors may require targeted practice and feedback.

5.3 Collaboration and Individual Accountability

Collaborative learning can provide opportunities to articulate reasoning and compare strategies. The research directions represented by Al-Hanifah et al. (2023) and Manik et al. (2020) make communication a relevant consideration when designing mathematical instruction.

However, collaboration involves a trade-off between shared learning and individual accountability. Stronger

students may dominate discussions, while less confident students may adopt correct answers without understanding them. Teachers can address this risk by assigning rotating roles, asking every student to explain one step, collecting individual written solutions, and using follow-up questions to test understanding.

Digital collaboration can extend opportunities for discussion, but the instructional benefits depend on access, task structure, and the quality of interaction. Online group work should therefore be evaluated through the mathematical reasoning it produces, not merely by the completion of shared activities.

5.4 Differentiation, Assessment, and Implementation Constraints

Students enter mathematics lessons with different levels of prerequisite knowledge, confidence, and experience. A uniform instructional sequence may consequently provide too little support for some students and insufficient challenge for others. Diagnostic assessment and flexible scaffolding are essential for maintaining an appropriate level of difficulty.

The discussion of self-assessment in Mulyono et al. (2018) suggests a relevant role for reflective learning. Nevertheless, self-assessment should complement rather than replace teacher evaluation. Students may overestimate their understanding or lack the vocabulary needed to describe their difficulties accurately. Clear rubrics, model explanations, and guided reflection can improve the usefulness of self-evaluation.

Implementation also requires time, appropriate resources, and teacher preparation. Contextual problems may take longer than direct explanation, and collaborative activities can introduce classroom-management challenges. Visual materials may be difficult to produce in resource-constrained environments. These constraints imply that the framework should be implemented selectively and proportionately rather than as a requirement to make every mathematics lesson complex or technology-intensive.

Outdoor learning and comic-assisted instruction are possible adaptations, but they should be selected according to the learning objective, student needs, and available resources (Arifani et al., 2021; Kusumawati et al., 2024). Instructional design should prioritize mathematical clarity over novelty.

5.5 Limitations and Future Research

The principal limitation is the conceptual nature of this study. The supplied references establish relevant research topics, but complete methodological details and empirical results were not available for critical appraisal. The synthesis therefore cannot establish whether the proposed six-phase sequence outperforms alternative instructional approaches.

A second limitation concerns the breadth of mathematical proficiency. Different age groups, mathematical topics, and educational contexts may require different instructional arrangements. A sequence suitable for introducing proportional relationships may need substantial modification for abstract proof, probability, or advanced algebra.

Future research should first develop and validate assessment instruments corresponding to the framework's five proficiency dimensions. Researchers could then conduct pilot studies to examine implementation fidelity, student engagement, and the feasibility of each instructional phase. Larger comparative studies could investigate changes in conceptual understanding, procedural fluency, problem-solving, reasoning, and transfer while accounting for baseline achievement and classroom differences.

Longitudinal research would be particularly valuable because immediate performance improvements may not indicate durable learning. Studies should examine whether students retain mathematical concepts, apply them to unfamiliar problems, and become more independent in selecting strategies. Researchers should also report null or mixed results to determine which elements of meaningful instructional design are beneficial under specific conditions.

Overall, the framework offers a structured research proposition: mathematical proficiency may be better supported when instruction deliberately connects understanding, application, communication, and reflection. Whether this coordination produces measurable advantages remains an empirical question rather than an established finding of the present study.

6. CONCLUSION

This paper developed a conceptual framework for

enhancing students' mathematical proficiency through Meaningful Instructional Design strategies. The framework integrates diagnostic assessment, contextual activation, knowledge construction, guided application, collaborative reasoning, and reflective evaluation. These phases are intended to connect instructional activities with conceptual understanding, procedural fluency, problem-solving, mathematical communication, and knowledge transfer.

The reference-based synthesis indicates the relevance of coordinating research directions concerning motivation, cognitive outcomes, mathematical literacy, reasoning, communication, contextual learning, and self-assessment. Its principal contribution is an instructional structure that translates these complementary dimensions into a sequence of teaching actions and observable assessment criteria.

The framework also emphasizes that engagement, contextual relevance, and collaboration cannot independently establish mathematical mastery. Teachers must evaluate students' individual reasoning and their ability to apply knowledge beyond familiar examples. Successful implementation will depend on appropriate scaffolding, instructional alignment, available resources, and the responsiveness of teaching practices to student needs.

Because the study is conceptual, the framework should not be interpreted as an experimentally validated intervention. Future research should test its feasibility and effectiveness using validated assessments, comparative study designs, and longer-term measures of retention and transfer. Such research could establish which components are most useful, for which students, and under what instructional conditions. The framework therefore provides a foundation for systematic instructional development and empirical investigation into meaningful mathematics learning.

REFERENCES

1. Al-Hanifah, J. A., Cholily, Y. M., & Ummah, S. K. (2023). Analysis of Students' Analytical Thinking Ability and Mathematical Communication Using Online Group Investigation Learning Model. *Mathematics Education Journal*, 7(1).
2. Anggraini, H. D. (2023). Pengaruh Model Pembelajaran Meaningful Instructional Design

- terhadap Motivasi dan Hasil Belajar Kognitif Siswa. *Educatoria: Jurnal Ilmiah Ilmu Pendidikan*, 3(2), 56–71.
3. Apriani, A., Prihatiningtyas, N. C., & Husna, N. (2023a). Pengaruh Model Pembelajaran Meaningful Instructional Design (MID) Terhadap Kemampuan Pemecahan Masalah Matematis Siswa Pada Materi Sistem Persamaan Linear Tiga Variabel. *Innovative: Journal Of Social Science Research*, 3(2), 12131–12144.
 4. Apriani, A., Prihatiningtyas, N. C., & Husna, N. (2023b). Pengaruh Model Pembelajaran Meaningful Instructional Design (MID) Terhadap Kemampuan Pemecahan Masalah Matematis Siswa Pada Materi Sistem Persamaan Linear Tiga Variabel. *Innovative: Journal Of Social Science Research*, 3(2), 12131–12144.
 5. Arifani, H., Wardono, W., & Cahyono, A. N. (2021). Mathematics Literacy Skill Based On Self-Directed Learning On Meaningful Instructional Design Based Outdoor Learning. *Unnes Journal of Mathematics Education Research*, 10(A), 26–31.
 6. Gazali, R. Y., & Atsnan, M. F. (2022). Implementation of contextual approach as meaningful mathematics learning. *Jurnal Inovasi Pembelajaran Matematika*, 1(1).
 7. Glean, G. R., Sidabutar, R., & Simarmata, G. (2023). The Effect of Meaningful Instructional Design (MID) Learning Model on Students' Mathematical Reasoning Ability on the Material of Arithmetic Rows and Rows in Class VIII SMP Negeri 2 Tebing Tinggi. *EduMatika: Jurnal MIPA*, 3(4), 83–93.
 8. Karim, A. (2022). Analisis Bibliometrik Menggunakan Vosviewer Terhadap Trend Riset Matematika Terapan Di Google Scholar. *Jurnal Riset Pendidikan Matematika Jakarta*, 3(2).
 9. Kusumawati, I., Firdaus, F. M., & Oktari, V. (2024). Implementation of Meaningful Instruction Design Model Assisted by Comic on Students Understanding of Multiplication. *Kreano, Jurnal Matematika Kreatif-Inovatif*, 15(1), 97–108.
 10. Manik, M., Saragih, S., & Zulkarnain, Z. (2020). Kemampuan Komunikasi Matematis Peserta Didik Melalui Pembelajaran Berbasis Masalah (PBM): Studi Quasi Eksperimen di SMA Negeri 1 Pangkalan Kerinci. *JURING (Journal for Research in Mathematics Learning)*, 3(1).
 11. Mulyono, M., Kartono, K., & Rosyida, M. D. N. (2018). Self-assessment on the achievement of the ability of mathematical proportional application in Meaningful Instructional Design (MID) learning viewed from student's learning style. *Unnes Journal of Mathematics Education*, 7(1), 39–47.